APPLICATIONS NOTE

Maxwell Technologies®
BOOSTCAP® Ultracapacitor
Cell Sizing

Document # 10073627

Rev 3
**Background**

An ultracapacitor’s voltage profile (voltage vs. time) has two components; a capacitive component, and a resistive component. The capacitive component represents the voltage change due to the change in energy within the ultracapacitor. The resistive component represents the voltage change due to the equivalent series resistance (ESR) of the ultracapacitor. Figure 1 illustrates these two components for a constant current discharge. A charge profile will be similar, but voltages will increase rather than decrease. In this document, all analysis will be based on discharges. For multiple step applications, analyze each charge or discharge step separately.

![Discharge Profile](image)

Figure 1: Constant current discharge profile

In this figure, the following variables are indicated:

- \( V_w \) = working voltage
- \( ESR \) = voltage drop due to ESR (see equation 3)
- Capacitive = voltage drop due to discharge of the capacitor (see equation 2)
- \( V_{\text{min}} \) = minimum voltage allowed by system (or minimum voltage occurring during discharge)
- \( t_d \) = discharge time

The capacitive component is governed by the equation;
\[ i = C \frac{dV}{dt} \]  
\text{Equation (1)}

where:
- \( i \) = current
- \( C \) = capacitance
- \( dV \) = change in voltage
- \( dt \) = change in time (time of discharge)

Rearranging equation (1) and solving for \( dV \):

\[ dV = i \frac{dt}{C} \]  
\text{Equation (2)}

The resistive component is governed by the equation:

\[ V = i \cdot R \]  
\text{Equation (3)}

where:
- \( V \) = voltage drop across the resistor
- \( i \) = current
- \( R \) = equivalent series resistance

The total voltage change when charging or discharging an ultracapacitor includes both of these components. Combining the capacitive and resistive components in equations (2) and (3):

\[ dV = i \frac{dt}{C} + i \cdot R \]  
\text{Equation (4)}

Let us briefly analyze the variables in this equation.

\( dV \) = the change in voltage during the discharge of the capacitor. This is determined by knowing the working operating voltage \( (V_w) \), and the minimum allowable system voltage \( (V_{\text{min}}) \).
should be the typical operating voltage at the beginning of a discharge. In some cases, this will be the maximum voltage of the system ($V_{\text{max}}$), but in other cases it will not. Also, be careful to determine that the minimum allowable system voltage is used, and not simply the lowest voltage at which the system currently operates. Often, the present minimum voltage specification isn’t limited by the system, but by a component which may be replaced by the ultracapacitor.

\[ i = \text{the current during the discharge of the capacitor. This calculation assumes a constant current during the discharge. Since most applications are approximately constant power, or some other varying current, use the average current for this value. This can be determined by calculating the current at maximum voltage ($I_{\text{min}} = \text{Power}/V_{\text{max}}$), and at minimum voltage ($I_{\text{max}} = \text{Power}/V_{\text{min}}$), and averaging the two values.}\]

\[ dt = \text{the duration (in seconds) of the discharge pulse.}\]

\[ C = \text{the capacitance of the complete ultracapacitor system at its operating point. This value will be based on the number of individual capacitors in series or parallel. For ultracapacitors in parallel, the capacitance is additive. For ultracapacitors in series, the capacitance is additive at 1/capacitance. The capacitance will also be affected by the duration of the pulse. Very short pulses will require decreasing the effective capacitance and is addressed in a separate document 1007234.}\]
To determine how many cells are required in series, divide the maximum application voltage, \( V_{\text{max}} \), by the maximum allowable cell voltage. The maximum allowable cell voltage is determined by life and temperature considerations. Nominally, this can be assumed to be 2.5 volts per cell.

The number of cells in parallel is determined after the first iteration of this calculation. If the first iteration indicates that there is inadequate capacitance for the application’s requirements, the capacitance and resistance can be changed by either putting more cells in parallel or by using larger cells. In some instances, using fewer series cells and choosing to operate the individual cells at higher voltages is an option. This is a trade-off of performance vs. life, since higher operating voltages decrease life. This trade-off must be done on a case-by-case basis.

\[
R = \text{the resistance of the complete ultracapacitor system. This value will be based on the number of individual capacitors in series or parallel. The greater the number of cells in parallel, the lower the resistance. The greater number of cells in series, the greater the resistance. Note that this is the opposite of how capacitance is calculated. The resistance will also be affected by the duration of the pulse. Very short pulses will require decreasing the effective resistance. This is addressed in a separate document #1007234. Determining the number of cells in series or parallel has been discussed previously.}
\]

\[
R_{\text{total}} = R_{\text{cell}} \times \frac{\# \text{ series}}{\# \text{ parallel}}
\]  

Equation (6)
**Determining Application Variables**

In analyzing any application, we first need to determine the system variables. From these, we determine the value of the variables required to solve Equation (4). We therefore need to gather the following information about the application.

- $V_{\text{max}} = \text{maximum voltage}$
- $V_w = \text{working operating voltage}$
- $V_{\text{min}} = \text{minimum allowable voltage}$
- $i = \text{current requirement}$

Or

- $P = \text{power requirement}$
- $t = \text{time of discharge (or charge)}$

The complete set of required information is listed here:

1) max voltage
2) min voltage
3) allowable voltage change during pulse
4) power or current
5) duration of pulse
6) temperature
7) repetition rate
8) required life

1 - used to determine # of series cells
1, 8 - used to determine cell voltage
2,3,4,5,6 - used to determine required cell size, or number of parallel cells
6 - determines ESR in cold temperature applications
7 - determines self heating from resistive losses (and therefore cooling requirements)
We can now proceed in one of two directions: we can determine performance based on a known capacitor size, or we can determine the required capacitor size to achieve a specific performance. We will start by determining performance based on a known capacitor size.

**Example Sizing Solution Based on Known Ultracapacitor**

Let us assume we have an industrial un-interruptible power supply (UPS) application requiring 10 kilowatts (kW) for 5 seconds. The system will normally operate at 56 volts, and can function on a voltage as low as 25 volts. The system will never experience greater than 60 volts.

**Step 1: Determine basic system parameters**

\[
\begin{align*}
V_{\text{max}} & = 60 \text{ volts} \\
V_w & = 56 \text{ volts} \\
V_{\text{min}} & = 25 \text{ volts} \\
\text{Power} & = 10 \text{ kW} \\
\text{time} & = 5 \text{ seconds}
\end{align*}
\]

**Step 2: Determine the values of the variables in Equation (4)**

\[
\begin{align*}
dV & = V_w - V_{\text{min}} = 56 - 25 = 31 \text{ volts} \\
i & = \text{average current} \\
i_{\text{max}} & = \frac{\text{Power}}{V_{\text{min}}} = \frac{10,000 \text{ watts}}{25 \text{ volts}} = 400 \text{ amps} \\
i_{\text{min}} & = \frac{\text{Power}}{V_{\text{max}}} = \frac{10,000 \text{ watts}}{56 \text{ volts}} = 179 \text{ amps} \\
i_{\text{avg}} & = \frac{(400+179)}{2} = 289 \text{ amps} \\
i & = 289 \text{ A} \\
dt & = 5 \text{ sec} \\
C & = \text{total stack capacitance}
\end{align*}
\]
\( V_{\text{max}} \) is defined as 60 volts. We assume that since the application is a UPS system, it will be operating at or near its maximum voltage most of the time, and that the customer requires at least 10 years life. Divide \( V_{\text{max}} \) by the cell voltage to get the required number of cells in series. For the MC series of cells the cell voltage is 2.7V:

\[
\begin{align*}
V_{\text{max}} &= 60 \text{ volts} \\
\text{Cell voltage} &= 2.7 \text{ volts} \\
\# \text{ of cells} &= \frac{60}{2.7} = 23 \text{ series}
\end{align*}
\]

From equation 5,

\[
C_{\text{total}} = C_{\text{cell}} \times \frac{\# \text{ parallel}}{\# \text{ series}}
\]

Assume use of the BCAP1500, cell capacitance = 1500 F (for a BCAP1500)

\# parallel = 1 (initially a single string)

\# series = 23

total stack capacitance = \( \frac{1800}{23} = 65.2 \text{ F} \)

From equation 6

\[
R_{\text{total}} = R_{\text{cell}} \times \frac{\# \text{ series}}{\# \text{ parallel}}
\]

\( R \) = total stack resistance. Since we have already selected a cell and the length of the series, use these figures to calculate stack resistance.

Cell resistance = 0.00047 ohm (for a BCAP1500)

\# series = 23

total stack resistance = 0.00047 ohm * 23 = 0.0108 ohm
Having all the variables defined, we can solve for the change in voltage (dV), or for duration (dt). Solving for a given change in voltage allows us to see how much margin we have on time. Solving for a given duration allows us to see how much margin we have on voltage. Since Equation (4) is already solved for dV, we will proceed in that direction.

\[
    dV = i \frac{dt}{C} + i \cdot R
\]

Substituting in the values for i, dt, C, and R;

\[
    dV = 289A \cdot \frac{5 \text{ sec}}{65.2 \text{ F}} + 289A \cdot 0.0108 \text{ ohm}
\]

\[
    dV = 22.16 + 3.12
\]

\[
    dV = 25.28 \text{ volts}
\]

Our original requirement allowed a voltage change of 31 volts, and the solution provides 25.3 volts, so this is a good fit. We have 81% of the allowed voltage drop (25.3/31). Since the equations are simple linear relationships, the optimum ultracapacitor would be 81% the size of a BCAP1500 or a 1223F capacitor.
Finding the Optimum Size Based on Unknown Ultracapacitor

An alternative method to size a solution is to determine the optimum size which meets the requirements, then adjust based on actual product offerings. This is a good method if one does not yet have the experience to make a good first estimate of appropriate size, as used in the previous example.

Step 1: Determine basic system parameters (same as previous example)

\[ V_{\text{max}} = 60 \text{ volts} \]
\[ V_w = 56 \text{ volts} \]
\[ V_{\text{min}} = 25 \text{ volts} \]
\[ \text{Power} = 10 \text{ kW} \]
\[ \text{time} = 5 \text{ seconds} \]

Step 2: Determine the values of the variables in equation #4

\[ dV = V_w - V_{\text{min}} = 56 - 25 = 31 \text{ volts} \]
\[ i = \text{average current} \]
\[ i_{\text{max}} = \frac{\text{Power}}{V_{\text{min}}} = 10,000 \text{ watts}/25 \text{ volts} = 400 \text{ amps} \]
\[ i_{\text{min}} = \frac{\text{Power}}{V_{\text{max}}} = 10,000 \text{ watts}/56 \text{ volts} = 179 \text{ amps} \]
\[ i_{\text{avg}} = \frac{(400+179)}{2} = 289 \text{ amps} \]
\[ i = 289 \text{ A} \]
\[ dt = 5 \text{ sec} \]
\[ C = \text{total stack capacitance} \]
\[ R = \text{total stack resistance}. \]

We will solve for the total stack capacitance by using the RC time constant for determining resistance. The RC time constant of an ultracapacitor is the product of its capacitance value
and resistance value. For this example, we will assume an ultracapacitor time constant of 0.7 seconds. Note that a BCAP1500 has 1500 F and 0.00047 ohms = 0.705 seconds. If this is not known, use 1 second and repeat the process below to converge on cell requirements.

Since \( R \times C = 0.7 \) seconds, \( R = \frac{0.7}{C} \)

Having all the variables defined, we will rearrange equation #4 and solve for \( C \):

**Equation (4) originally:**

\[
dV = i \frac{dt}{C} + i \times R
\]

Substitute \( R = \frac{0.7}{C} \):

\[
dV = i \frac{dt}{C} + i \times \frac{0.7}{C}
\]

Factor out \( \frac{i}{C} \)

\[
dV = \frac{i}{C} \times (dt + 0.7)
\]

Solving for \( C \)

\[
C = \frac{i}{dV} \times (dt + 0.7)
\]

Substituting in the values for \( dV \), \( i \), and \( dt \);

\[
C = \frac{289A/31V}{(5 + 0.7)}
\]

\[
C = 53.1 \text{ F}
\]

Remember, this value of capacitance is the total stack capacitance. We must now determine the required cell capacitance. For this stage in the analysis, we only need to know the number of
series cells required. The analysis is the same as in the previous example, so we need 23 cells in series.

From equation 5,

\[ C_{\text{total}} = C_{\text{cell}} \times \frac{\# \text{ parallel}}{\# \text{ series}} \]

Setting \# parallel = 1

\[ C_{\text{total}} = \frac{C_{\text{cell}}}{\# \text{ series}} \]

Solving for \( C_{\text{cell}} \)

\[ C_{\text{cell}} = C_{\text{total}} \times \# \text{ series} \]

Stack capacitance = 53.1 F
\# series = 23 cells
Cell capacitance = 1222 F

Our previous analysis indicated that a BCAP1500 allowed a voltage drop that was 81% of the allowed drop. It was noted that the optimum ultracapacitor size would be 81% of a BCAP1500. Note that 81% of 1500 F is 1222 F.

If this were our initial analysis and we determined a 1222 F cell was the optimum solution, we would then look to the actual product offerings.